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MULTIDISCIPLINARY OPTIMIZATION FOR ENGINEERING SYSTEMS:
ACHIEVEMENTS AND POTENTIAL

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MULTIDISCIPLINARY OPTIMIZATION FOR ENGINEERING SYSTEMS: ACHIEVEMENTS AND POTENTIAL

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ABSTRACT

The currently common sequential design process for engineering systems is likely to lead to suboptimal designs. Recently developed decomposition methods offer an alternative for coming closer to optimum by breaking the large task of system optimization into smaller, concurrently executed and, yet, coupled tasks, identified with engineering disciplines or subsystems. The hierarchic and non-hierarchic decompositions are discussed and illustrated by examples. In conclusion, an organization of a design process centered on the non-hierarchic decomposition is proposed.

INTRODUCTION

There is a growing realization that in complex engineering systems the mastery of the interactions among the disciplines and subsystems is as important for successful design as technologies used in any individual discipline or subsystem. Examples abound in nuclear industry, advanced ship-building, and automobile manufacturing, with a classic case provided by hypersonic aircraft design. Unlike in a conventional transport aircraft where optimal use of such interactions could make a difference between a very good and merely good vehicle performance, in hypersonic aircraft it may make the difference between flying and staying on the ground.

Early attempts to solve the problem by wrapping an optimization loop around a set of computer programs corresponding to the governing disciplines proved disappointing (ref.1) for reasons clear in retrospect. That approach tended to exclude the human intellect from the process, and the computational time and cost of repeated executions of coupled disciplinary analyses was prohibitive. Most importantly perhaps, the approach disregarded the engineers' thoroughly practical desire to form specialty groups, each group assuming responsibility for part of the design problem in exchange for a professional independence in the choice of means to do the job. It appears that the lag in large scale optimization applications behind the progress in optimization theory observed, for example, in a survey given in ref.2, may be attributed, partly, to the shortcomings of the above approach.

Stimulated by a realization that a different approach is needed, efforts have recently been increasing to develop methods that would bring to the entire design process the same mathematical efficiency, consistency, and rigor that have been achieved by computational methods in the contributing engineering disciplines while allowing a specialist to retain responsibility for a part of the entire task within his domain. The intuitively obvious and well-established practice of breaking a large task into smaller ones, together with an array of mathematical methods reviewed in ref.3, form a basis for a new approach that has begun producing a new methodology and a growing application experience, e.g., ref.4, and ref.5.

This peper outlines two algorithms for bringing optimization into engineering system design: the hierarchic decompcsition and non-hierarchic decomposition, originally introduced in ref.6, and ref.7, ref.8, respectively. A hierarchic decomposition leads to separate optimizations, supported by their own sensitivity analyses, for each part of the system. The non-hierarchic method applies decomposition to the system analysis and sensitivity analysis only, that is to the part of optimization

responsible for more than 90% of the total cost in large system applications. The optimization itself remains undivided but becomes linearized so that it may be effectively solved for a very large number of design variables.

The two methods are presented as alternatives to the currently prevalent sequential decision making in design which is shown, according to ref.9, as leading to suboptimal results. They are illustrated by application examples.

SUBOPTIMALITY IN SEQUENTIAL DESIGN PROCESS

The prevalent practice in today's design process is to make major decisions sequentially. An exception is the early conceptual stage where the major disciplines are given simultaneous consideration but the analyses are simplified and, therefore, may not reliably identify an optimal design. After that stage the process settles into a historically evolved sequence illustrated in Fig.1 for aircraft as an example. The boxes in the figure symbolize major disciplines, the inner loops in each box stand for iterative disciplinary optimizations (judgmental and formal) that manipulate local design variables toward betterment of some disciplinary objective such as minimum weight within constraints. The outer loops linking the boxes imply interdisciplinary optimizations toward improving the aircraft performance within constraints. For graphic simplicity the partial overlap of the boxes in time that usually takes place is not shown.

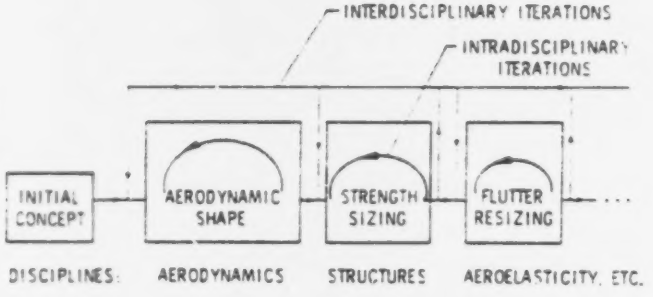

Figure 1. Sequential design process.

Critique of the process depicted in Fig 1. is given in ref.9 asserting that it must lead to suboptimal results. The assertion is based on the premise that the time and budget constraints on the process usually render the outer optimization loops impossible, or at least impractical, to execute. In the absence of the system level feedbacks, design decisions made upstream restrict the design freedom downstream. The result must be an underperforming design, as shown in ref.9 using a generic example illustrated next.

The example (Fig.2) is that of a hypothetical aircraft whose design optimization is reduced for the discussion purposes to a plot of the contours of a performance measure, P, e.g., payload for a given range, and two performance constraint boundaries (infeasible side cross-hatched), C1 and C2, e.g., take-off field length and the rate of climb, as functions of the wing aspect ratio, AR, and the wing structural minimum weight. The aspect ratio is one of the design variables typically set early in the process (Fig.1, AERODYNAMICS), primarily on the basis of aerodynamic considerations. The minimum structural weight may be regarded as a synthetic measure of a multitude of cross-sectional sizing variables decided in the discipline of structures (Fig.1, STRUCTURES).

By inspection, the point O1 is the constrained maximum of P. However, as the process in Fig.1 continues additional constraints may be found critical, for instance, a flutter speed constraint, C3,

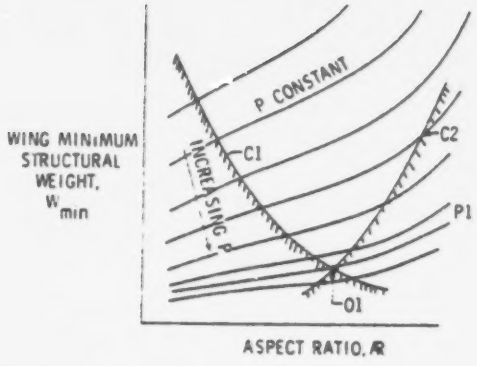

Figure 2. Example of an aircraft design design space with constraints.

shown in Fig.3. The added constraint invalidates the previous optimum at O1 and establishes a new one at O3. However, in the sequential process the AR may have already been frozen (the aircraft configuration decided) so that the only design freedom still available to deal with the additional constraint is to resize (or to mass balance) the structure paying a weight penalty represented by moving from O1 to O2 in Fig.3. The point O2 corresponds to a new design located on a P contour lower than the one passing through point O3. The difference of the P value between the contours passing through the points O2 and O3 measures the penalty loss relative to the performance that would be attainable if the AR was still available as a design variable at the time when the additional constraint was discovered.

The performance loss is relative - the performance at the point O2 may still be very good but not as good as it might have been if one of the design variables were not eliminated. It is in this relative sense, or in the sense of the potential left unused, that the design obtained in a process of sequential elimination of design variables must be suboptimal.

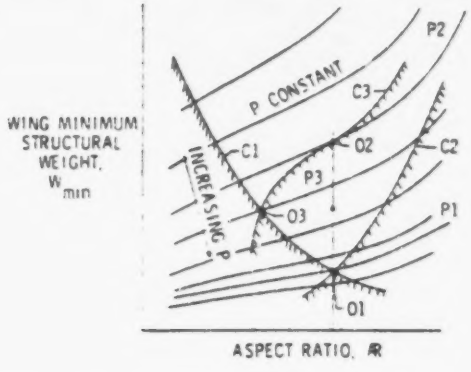

Figure 3. Adding another constraint.

The suboptimality is a symptom of what in ref.9 is termed the paradox of the sequential decision making in design. The paradox stems from the disparity, illustrated in Fig.4, between the accumulation of the knowledge about the object of design (vertical axis), brought about by the analysis and experimentation, and the gradual reduction of the design freedom (vertical axis), resulting from freezing of the design variables, that occur as the design process progresses in time (horizontal axis). The paradox is that the knowledge underlying the design increases but the ability to act on that knowledge decreases.

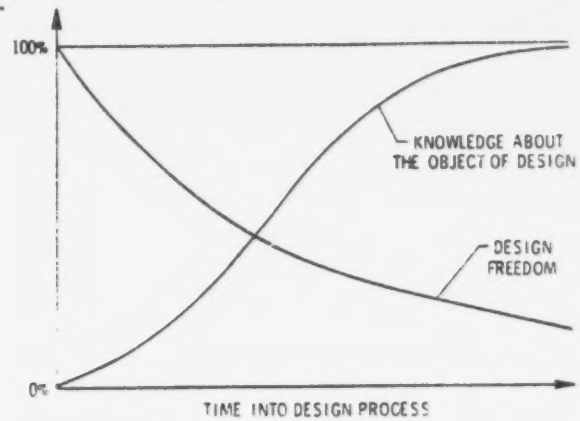

Figure 4. Paradox of the sequential design.

IMPROVING THE DESIGN PROCESS BY DECOMPOSITION

One remedy to the paradoxical situation depicted in Fig.4 is to make the knowledge curve rising steeper in order to have more information for acting on when the design freedom is still high. The well-established practice strives to accomplish this at the early design stages by generating information about the major aspects of the problem very rapidly but at the price of using superficial, simplified analysis, e.g., lifting line theory and statistical structural weights in aerodynamics and structures, respectively. This approach, enhanced with human judgment, is quite adequate in closing the inner and outer feedback loops shown in Fig.1 in conventional projects well grounded in the past experience. However, its reliability is questionable in far-out projects, e.g., a hypersonic aerospace plane, or a space station, for which such experience is lacking and must be made up by increased depth of analysis.

Rapid progress in computer technology provides increasingly powerful means for bringing deeper analysis into earlier stages of design but, obviously, there are limits to the knowledge curve steepness. Therefore the growing importance of the other remedy of retaining more design freedom at the later design stages - that is making the design freedom curve flatter.

HIERARCHIC DECOMPOSITION

One of the means for the above is a hierarchic decomposition which applies if the system can be divided into a set of "black boxes" forming a hierarchy shown in Fig.5. The "black boxes" represent either the physical subsystems, e.g., aircraft structure and engine, or the disciplines, e.g., aerodynamics and structural mechanics, and in both cases, for discussion purposes, the "black box" is simply a data converter that transforms input into output, e.g., load, geometry, material input data to displacement and stress output data in structural analysis. It is the input/output data flow

that determines the type of decomposition. In a hierarchic decomposition depicted in Fig.5, that flow is vertical, no data are transmitted between a pair of "black boxes" located at the same decomposition level.

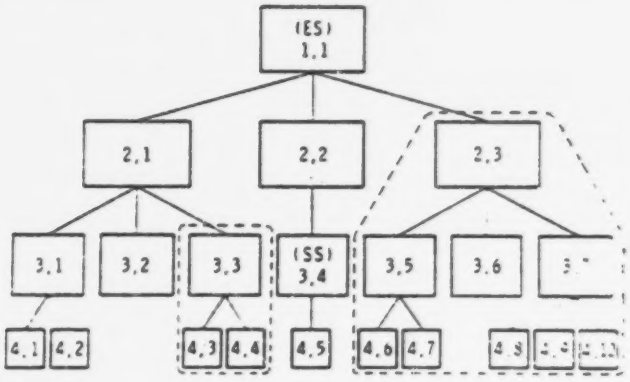

Figure 5. Hierarchic decomposition of a system.

Multilevel Optimization by Hierarchic Decomposition

A multilevel optimization method defined in ref.6 exploits the above hierarchy by transmitting the analysis results in the top-down direction and the optimization results in the opposite direction. Let us define U as the output vector from a particular "parent" and V as the input vector into that parent's "child" at the lower level. In a hierarchy comprising more than two levels, the child is a parent to a child at the next lower level, and so on, recursively. Then, U becomes V in a recursive, "parent-to-child" progression of analyses that extends top-down to the lowest level. The optimization operation begins at that level and flows upward subjecting each black box to a separate optimization using its own, unique, vector of design variables. The design variable vectors are recursively named for each parent-child pair: X in the parent and Z in the child. The analysis of the parent defines its U as a function of X, U = U(X).

A child optimization executes using design variables Z for a particular constant V = Vo whose elements become the optimization parameters. Consequently, the optimal values of F and design variables Z can be written as:

$$F_{opt} = F_{opt}(V); \quad Z_{opt} = Z_{opt}(V); \tag{1}$$

These functions are, of course, not available in an explicit analytical form but their derivatives dFopt/dVi and dZopt/dVi, called the derivatives of optimum with respect to parameters, may be obtained by means of an algorithm introduced in ref.10 and ref.11. These derivatives are a key element of the decomposition because they make it possible to approximate the functions Fopt(V) and Zopt(V) by a linear extrapolation

$$F_{opt} = F_o + \frac{dF_{opt}}{dV_i}(V_i - V_{io}); \tag{2}$$

$$Z_{opt} = Z_o + \frac{dZ_{opt}}{dV_i}(V_i - V_{io}); \tag{3}$$

where Fo and Zo stand for Fopt and Zopt obtained for constant values of parameters Vo.

Moving now from a child to its parent black box optimization, one may use eq.2 and 3 and the relations

$$V = U(X); \quad V_o = U_o; \tag{4}$$

substituted into eq.2 and 3 to approximate the Fopt and Zopt as functions of X by extrapolation

$$F_{opt} = F_o + \frac{dF_{opt}}{dU_i}(U_i(X) - U_{io}); \tag{5}$$

$$Z_{opt} = Z_o + \frac{dZ_{opt}}{dU_i}(U_i(X) - U_{io}); \tag{6}$$

The coupling represented by the chain of eq. 1 through 6 is recursive in the sense that it applies to any parent-child pair throughout the hierarchy. Further details are available in ref.6 but the key point is that these recursive relations transmit the information about the effect of the higher level design variables on the lower level objective and variables to the very top of the hierarchy (Fig.5). That means that the optimization in the top black box representing the entire system may be performed with a limited set of the system level design variables and, yet, it will be sensitive to the influence of these variables on every black box making up that system.

The above hierarchic decomposition was referred to in ref.6 as a linear decomposition because of its dependence on linear extrapolations (eq.2, 3, 5, and 6) and was demonstrated in ref.12 on an example of a framework structure. It was also used as a basis to formulate an algorithm for a structural optimization by substructuring in ref.13.

Applications

A multidisciplinary application of the method, reported in ref.14, involved optimization of a wing of a passenger transport aircraft of a wide body class shown in Fig.6 (left) and proved effective in handling a very large number of design variables and constraints in a problem that required computationally expensive analyses. The optimization objective was to minimize the fuel consumption for a typical mission, while satisfying the constraints drawn from the disciplines of structures, aerodynamics, and performance, the respective examples being stresses, displacements, transonic wave drag rise, take-off roll length, and range.

The system decomposition resulted in a hierarchy depicted in Fig.7 with the aircraft performance analysis and optimization, including aerodynamics, at the top level, structural analysis and optimization at the middle level, and the individual wing cover panel analyses and optimizations (total of 316 panels) represented at the bottom level. Analysis depth was characterized by the use of: an energy-based flight mechanics in the performance evaluation, a semiempirical drag calculation and the Computational Fluid Dynamics (CFD) methods in aerodynamic analysis, a finite element model shown in Fig.6 (right) in the wing box analysis, and closed form expressions for evaluation of local buckling in the individual cover panels. The corresponding design variables included the airfoil relative thickness (depth-to-chord ratio) at the top level, stiffened wing cover equivalent thickness distribution at the middle level, and the detailed cross-sectional dimensions of the wing cover skin reinforced by stringers at the bottom level. The fuel consumption was the objective function in the top level optimization. The purpose of each of the middle and bottom

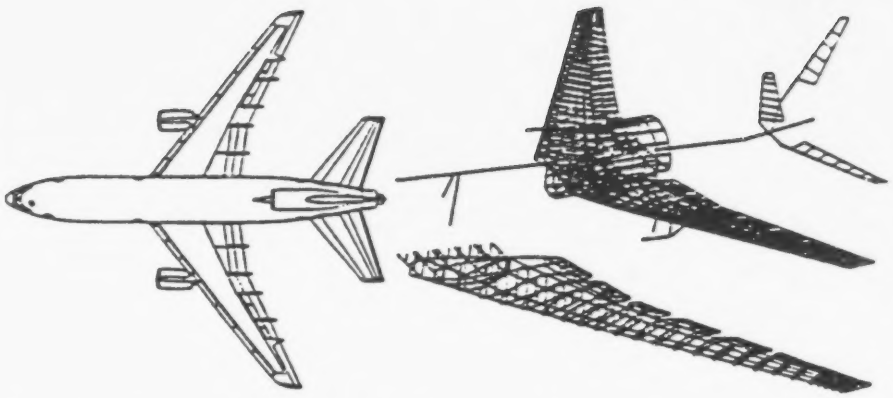

Figure 6. A transport aircraft and its finite element model.

level optimizations was to minimize the constraint violation (maximize the constraint satisfaction). For this purpose, all the constraints in a black box were represented by a single function called the cumulative constraint which was used as the objective. Nonlinear mathematical programing (NLP) was the optimization tool at every level. Examples of the analysis output transmitted from the top down and the optimization information passed from the bottom up are given in Fig.7 keyed to the numerals inscribed by the arrows.

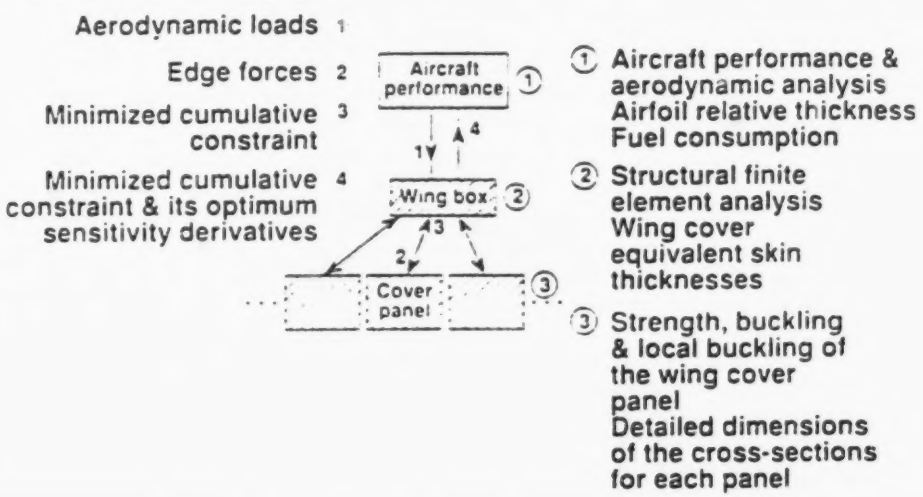
Aerodynamic loads 1
Edge forces 2
Minimized cumulative constraint 3
Minimized cumulative constraint & its optimum sensitivity derivatives 4
Aircraft performance
Wing box
Cover panel
① Aircraft performance & aerodynamic analysis
Airfoil relative thickness
Fuel consumption
② Structural finite element analysis
Wing cover equivalent skin thicknesses
③ Strength, buckling & local buckling of the wing cover panel
Detailed dimensions of the cross-sections for each panel

Figure 7. Example of a hierarchic, three-level decomposition of aircraft system

A sample of the voluminous results given in ref.14 is illustrated in Fig.8. It shows the method converging at a rather rapid rate to the same end state from two deliberately varied starting points.

The closeness of the end state to the actual aircraft design that was extensively optimized by other means validated the present method.

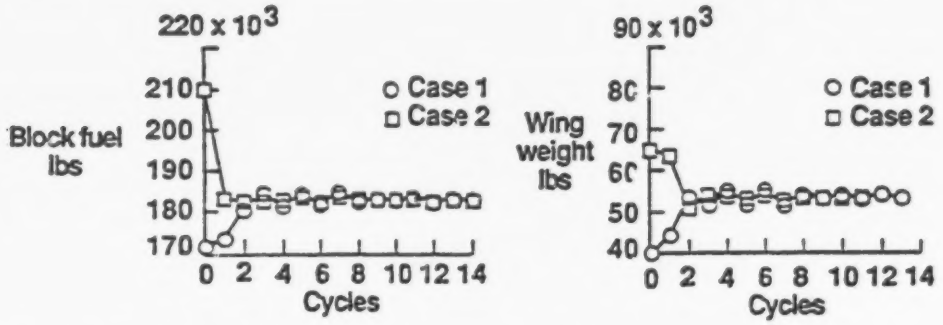

Figure 8. Histograms of three-level optimization of a transport aircraft; cases 1 and 2: initial design infeasible and feasible, respectively.

The above application broke new ground on three accounts. Firstly, it established that optimization with a large number of design variables, 1300+, and constraints, 1900+, involving a computationally intensive analysis (a finite element model of 1500+ elastic degrees of freedom) could be effectively solved using decomposition to break the large single NLP task of 1300+ design variables into a set of smaller NLP tasks none of which exceeded 7 design variables. Without decomposition the problem computational size was far beyond the bounds commonly regarded as practical. Secondly, it showed that a design detail (structural sizing) may be mathematically linked to the system performance (aircraft fuel consumption and flight constraints), with the linkage spanning several disciplines (structures, aerodynamics, aircraft performance). Previously, that linkage has never been available at the level of analysis used in ref.14. Finally, it developed a parallel organization of separate optimization tasks at the bottom level that could have been executed on concurrently operating computers (a single computer was used in the study). The amenability to concurrent processing that in hierarchic decomposition extends, in principle, to all levels except the top one as one may see in Fig.5, makes the method inherently compatible with the modern technology of distributed computing and with the natural human organization of specialty groups.

Determining a System Decomposition.

A prerequisite to optimization by decomposition is development of a hierarchy such as the one shown in Fig.5. This implies that the entire task at hand is "granular" - naturally separable into subtasks - and that each subtask falls into its place in the hierarchy. The granularity is usually obvious, suggested by the existence of specialty groups and by major analysis tools each forming a core of a subtask, and by division of the object of design into the physical subsystems. Each subtask proper place in the hierarchy is obvious in the design projects firmly rooted in the past experience, as was the airliner study in ref.14, but it may not be so obvious in attempting a new, unprecedented design such as an actively controlled space structure described in ref.15. The design called for structural dimensioning of the lattice column seen in Fig.9 protruding out of the Space Shuttle Orbiter and for a synthesis of the control system limiting the lattice column deformations.

In this case, the hierarchic decomposition scheme was built using a formal method which worked as follows. A set of N candidate modules (subtasks) are identified and placed, first in a random order, as square boxes on the diagonal of a diagram shown in Fig.10. The diagram is called the

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Nsquare diagram and under its formalism each square box horizontal sides represent its input ports while the output ports are represented by the vertical sides. A transmission path of an output from a module to its successor module is symbolized by a horizontal line emanating to the right from the right hand side of the box representing the source module. At a dot marking an intersection with a vertical line the data transmission path turns vertical and continues to a receiving module. Such data connections, called the feedforward data paths, identify for each source module one or

Figure 9. A flexible, actively controlled space structure.

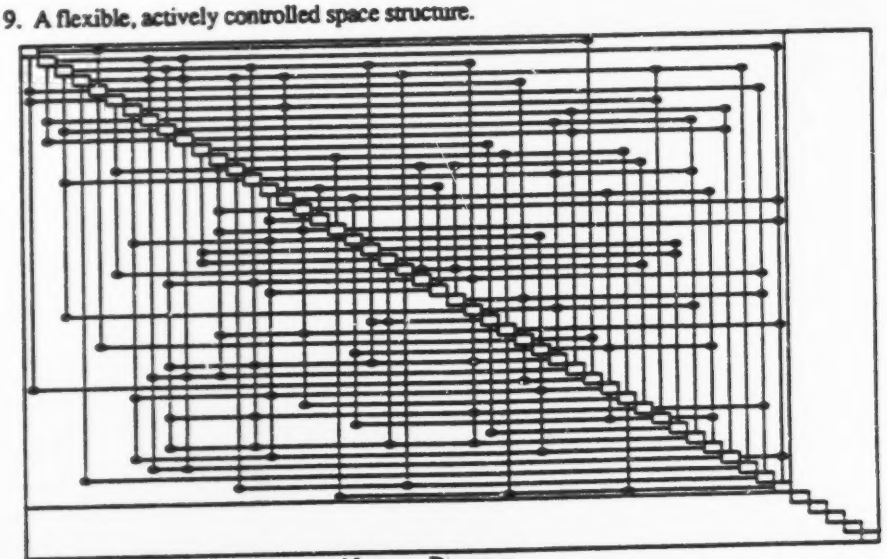


Figure 10. Modules randomly strung in a Nsquare Diagram

more successor modules that execute after the source module. There are also instances when a module sends data back to one or more of the preceding modules along the feedback paths that can be seen below the diagonal in the diagram.

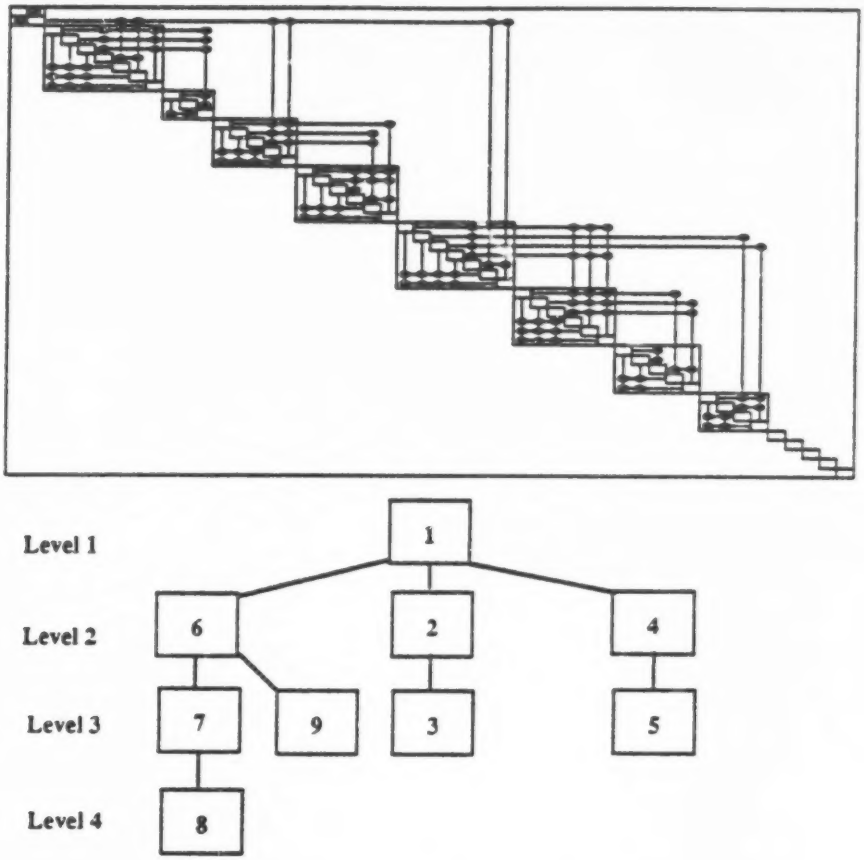
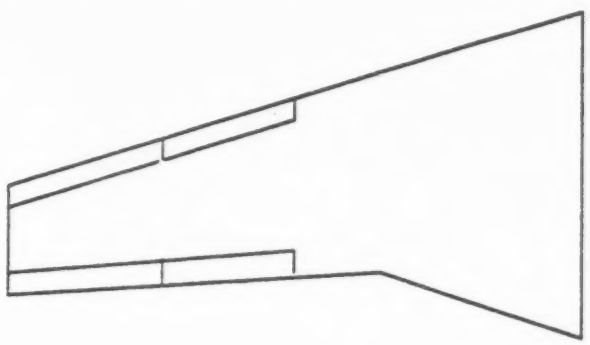


Figure 11. Modules organized in clusters (top) forming a hierarchic system (bottom).

For a set of modules randomly grouped as in Fig.10 no particular decomposition organization is visible but one may identify a hierarchic decomposition for it by means of a computer-aided formal procedure described in ref.16. The procedure systematically permutates the rows and columns in the Nsquare diagram, driven by rules that incorporate the principle of hierarchic decomposition according to which no module may send or receive data from another module at the same level. A typical result from ref.16 is shown in Fig.11 (top). It shows the modules regrouped so that the occurrences of the feedback have either been eliminated or limited to the clusters which, themselves, are linked only by the feedforward data paths. The clusters may now be represented as black boxes in a hierarchic decomposition comprising four levels as illustrated in Fig.11 (bottom). The modules inside a cluster constitute a non-hierarchic system discussed next.

NON-HIERARCHIC DECOMPOSITION

Some engineering systems cannot be decomposed into a purely hierarchic pyramid of modules because no reshuffling of the modules in the Nsquare diagram can eliminate information transmission links among modules at the same level (lateral links). Such systems are referred to as network systems (NS); a flexible wing with a pair of active control surfaces at the leading and trailing edge, Fig.12 (top), described in ref.17 is an example. The modules representing the wing are Aerodynamics, Structures, and Controls. They are coupled by information links defined in the diagram in Fig.12 (bottom). There are no rational reasons for placing any of these modules above the others in a hierarchy and none of the links may be severed, so the three modules must be treated as forming a one-level, non-hierarchic, coupled system. In a system of this type, changing a design variable that directly affect only one part of the system may have indirect, but significant, repercussions throughout. To account for that effect a method has been developed in ref.7 for calculation of the system behavior sensitivity with respect to design variables to guide the design decisions.



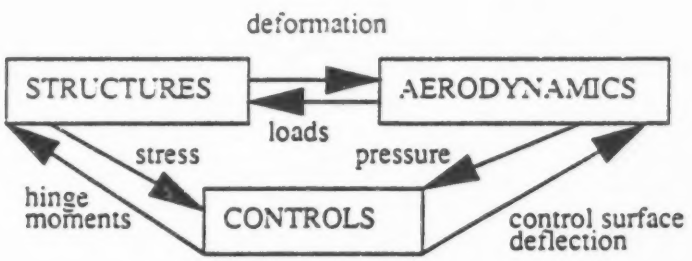


Figure 12. Actively controlled, flexible wing (top)and its system representation (bottom)

System Sensitivity Analysis

The wing in Fig.12 is an example by which to introduce the system sensitivity analysis from ref.7. The example has only three modules but that is enough to see a pattern that can be extended to any number of modules. The modules portrayed in Fig.12 (bottom) are regarded as input-to-output converters, and are labeled A, S, C for Aerodynamics, Structures, and Controls, respectively. Their respective output vectors of the behavior variables are Ya, Ys, and Yc. Input into a module

comprises the design variables Xi (elements of vector X) and the elements of vectors Y that may be cross-fed from other modules. From a mathematical viewpoint, a module is a set of equations which, when satisfied, yield a null right-hand-side. Together, the equations corresponding to each module in the system form a set which is internally coupled by the output-to-input cross-feeding.

The set of equations governing the entire system may be written as:

$$\begin{aligned} A((X, Y_s, Y_c), Y_a) &= 0; \\ S((X, Y_a, Y_c), Y_s) &= 0; \\ C((X, Y_s, Y_a), Y_c) &= 0; \end{aligned} \tag{7}$$

The list of arguments for each vector function shows the vector of unknowns (output) last and the input vectors grouped in the inner parentheses. The presence of the elements of the X and Y vectors in that inner parentheses is selective - not every element of X enters every module as input, and the same applies to Ya, Ys, and Yc. The number of unknown elements of the vectors Ya, Ys, and Yc must equal the number of equations represented by the corresponding vector functions A, S, and C. The equations may be nonlinear so that their solution may require an iterative algorithm. Examples of the elements Ya, Ys, Yc are aerodynamic pressure coefficients, structural displacements, angles of the control surface deflections, respectively. For elements of X one may mention the wing airfoil geometry variables and the wing planform aspect ratio, the wing structure cross-sectional dimensions, and the coefficients (gains) in the control law. The vector functions A, S, C may be implemented, respectively, as a CFD computer program, a finite element analysis program, and a control system analysis program.

Solution Ya, Ys, Yc of eq.7 describes a behavior of the system for a given X and the object of the sensitivity analysis is to obtain the total derivatives, dY/dXi, of the vectors Y with respect to design variables X, referred to as the system sensitivity derivatives. Finite differencing to obtain these derivatives is impractical for large systems for the reasons of computational cost, potentially poor accuracy, and organizational inertia (each module may be operated by a separate group of specialists). To bypass these difficulties, a new algorithm for system sensitivity analysis was introduced in ref.7.

The algorithm calculates the system sensitivity derivatives from a set of equations derived from the implicit function theorem. Regardless of the nature of eq.7 (nonlinear, transcendental, etc.), the sensitivity equations are always linear, algebraic, simultaneous equations:

$$[M]\ \frac{dY}{dX_i} = \frac{\partial Y}{\partial X_i} \tag{8}$$

where the matrix of coefficients M is composed of the diagonal identity submatrices and off-diagonal submatrices of the partial sensitivity derivatives (the Jacobian matrices):

$$[M] = \begin{bmatrix} I & -J_{as} & -J_{ac} \\ -J_{sa} & I & -J_{sc} \\ -J_{ca} & -J_{cs} & I \end{bmatrix} \tag{9}$$

To illustrate the meaning of the off-diagonal matrices, the Jas is a Jacobian of the partial sensitivity derivatives - an Na*Ns matrix - of the Na pressure coefficients output from A with respect to the Ns wing structural deflections input into A from S. The i-th column of Jas comprises the partial derivatives with respect to the j-th displacement.

The right-hand-side vector is composed of the partial derivatives of the outputs Ya, Ys, and Yc with respect to one particular design variable, so that the solution vector dY/dXi contains

derivatives of the coupled system A, S, C with respect to that variable. For many design variables, eq.8 may be efficiently solved with many right-hand-sides by generating and factoring the matrix M only once and, then, backsubstituting each rhs vector over the factored M.

The partial derivatives in M and the rhs vector are, by definition, calculated as a separate task for each module, A, S, C by any specialized disciplinary method, including semi-analytical algorithms, finite differencing, and even experiments. Resorting to finite differencing in this task is still advantageous comparing to finite differencing on the entire system analysis. Among the major engineering disciplines, the theory and practice of sensitivity analysis is the most advanced in structures (ref.18) but has begun taking hold in other disciplines as well (ref.19). In aerodynamics, ref.17 demonstrated feasibility of finite differencing in a large application, and a semi-analytical approach was formulated in ref.20.

The system sensitivity derivatives, dY/dXi, obtained from eq.8 fully account for all the couplings in eq.7. As the number of modules in eq.7 increases, the dimensionality of eq.8 increases accordingly but M tends to become block-sparse because, usually, not every module is linked to every other one and wherever a link is missing so is the corresponding off-diagonal Jacobian. Similarly, the rhs vector for a particular design variable, Xi, has null elements wherever that variable does not directly affect the vectors Y. As pointed out in ref.7, the dimensionality of eq.8 critically depends on how many output elements are cross-fed to input among the modules. For the method to be practical in large applications, one may have to carefully limit the number of such cross-fed elements. For instance, the wing finite element analysis in S may output thousands of displacements. However, to capture the elastic deformation effect on the aerodynamic loads computed in A, one may condense the deformation information by, say, using only a few displacement functions whose amplitudes are input into A as an update on the deformed wing shape. Then, the partial derivatives of the pressure coefficients need to be computed with respect to only those few amplitudes instead of the thousands of the finite element model displacements.

System Sensitivity Applications Examples

To close the discussion of the system sensitivity, a few examples for dY/dXi are in order. The derivative of the wing drag coefficient with respect to a cover panel skin thickness is an example of a quantity that depends on the aerodynamic loads-structural deformations coupling. The corresponding partial derivative is zero. The same derivative with respect to a particular coefficient in the control law is another example that involves interaction of all three modules in the system shown in Fig.12 (bottom). Again, the corresponding partial derivative is zero. However, the drag coefficient derivative with respect to the wing sweep angle exists both as a partial derivative $\partial Y/\partial X_i$ - the angle directly affects aerodynamics - and as a total derivative dY/dXi reflecting the interaction of all three modules, so that, in general, the partial and total derivative values are different. Finally, if the system from Fig.12 was augmented by the aircraft performance analysis module, one could extend the eq.8 pattern to calculate such performance derivatives as range or payload with respect to the wing aspect ratio, accounting for the trade-off of the structural weight vs. the aerodynamic drag of a flexible wing with active control.

The strength of the couplings of the modules in a system may create a drastic difference between the partial and total derivatives as shown in Fig.13 from ref.21. The ordinate is a flexible wing trimmed angle of attack defined as the incidence angle of the wing root chord needed to generate a specified amount of lift. The abscissa is the wing forward sweep (negative degrees). The graphs for the rigid and flexible wing are marked with squares and circles, respectively. The trimmed angle of attack derivatives at an arbitrary value of 20 degrees of the sweep angle are visualized by the slopes of the tangents. In this system of two interacting disciplines - Aerodynamics and Structures - the partial derivative predicted by Aerodynamics only computed for the rigid wing differs not

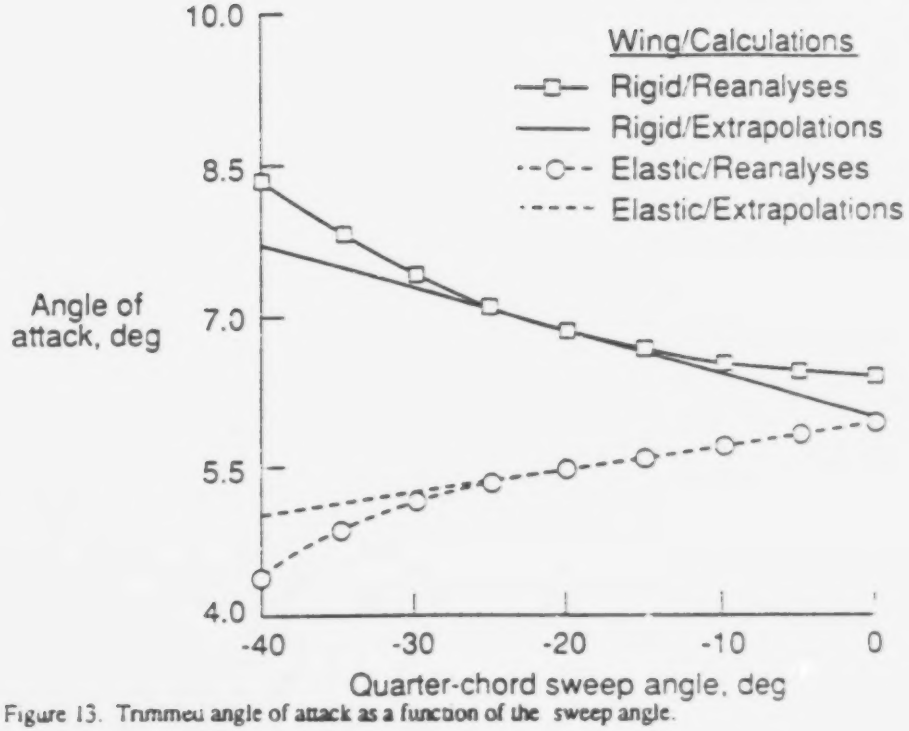


Figure 13. Trimmed angle of attack as a function of the sweep angle.

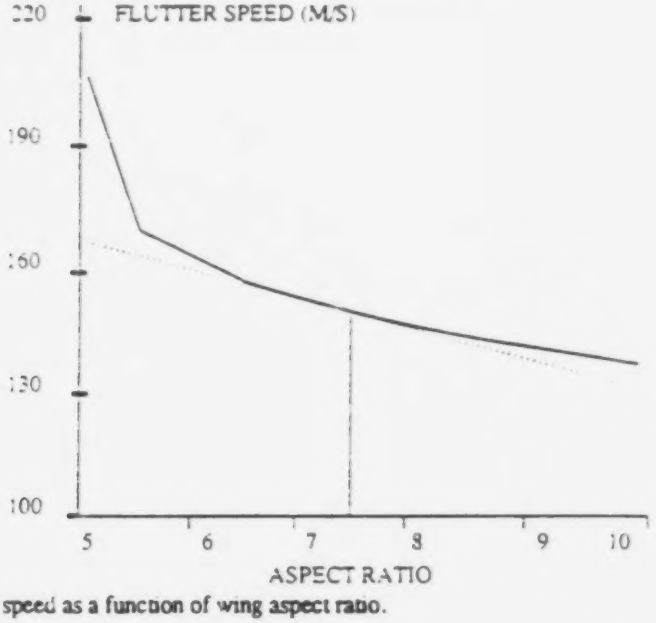


Figure 14. Flutter speed as a function of wing aspect ratio.

only in value but also in sign from the total derivative obtained for the flexible wing. In such a case, reliance on the partial derivatives may completely misguide design decisions.

The usefulness of the system sensitivity derivatives as a guide in the design process depends on the nonlinearity of the system behavior. Strong nonlinearity compels one to proceed in small steps, frequently updating the function and derivative data by re-analysis. Flutter sensitivity results reported in ref 22 are encouraging in this regard since they show flutter speed functions as smooth, and only mildly nonlinear over broad intervals of the design variables despite complex interaction of structural dynamics and unsteady aerodynamics. A typical example is reproduced in Fig.14. Results of a similar nature were given in ref.17 for a flexible, actively controlled wing such as the one shown in Fig.12.

Sensitivity Analysis vs. Parametric Study

Since in current engineering practice, sensitivity information is usually sought by parametric studies, it will be useful to compare such studies with formal sensitivity analysis. A typical parametric study for an example of a propeller would call for repeated analysis to generate the propeller efficiency data points to which one may fair a curve as in Fig.15 (left) for the diameter as one design variables. The plot shows the nature of the function at a glance in the entire interval of interest and reveals the extrema, providing instant insight. However, if there are more design variables, for instance, the blade pitch and taper, the number of curves to look at quickly escalates combinatorially beyond the limits of human comprehension, and the attendant analysis cost also becomes excessive.

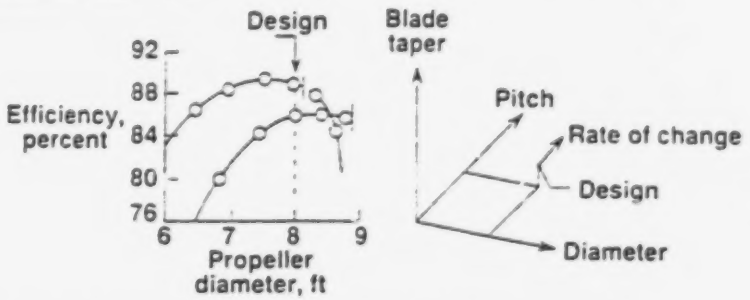


Figure 15. Typical parametric study results vs. sensitivity analysis results.

On the other hand, sensitivity derivatives of the propeller efficiency with respect to the three design variables mentioned above may be computed for a single design point (a setting of all the variables) for all the variables at a cost that does not explode combinatorially with their number. The derivatives may be interpreted as the components of the propeller efficiency gradient vector (maximum rate of change), shown in Fig.15 (right). The vector points the direction of the efficiency increase and may be used as input into a formal optimization algorithm. However, the vector carries only a local information and the overall shape of the function remains hidden to be determined only by a step-by-step exploration in the pointed direction. Thus, the two methods provide different types of information and complement each other.

System Sensitivity Analysis Applied to Entire Aircraft

The system sensitivity analysis algorithm outlined above may be applied to an entire aircraft as pointed out in ref.8. For example, consider a typical commercial, subsonic transport aircraft design

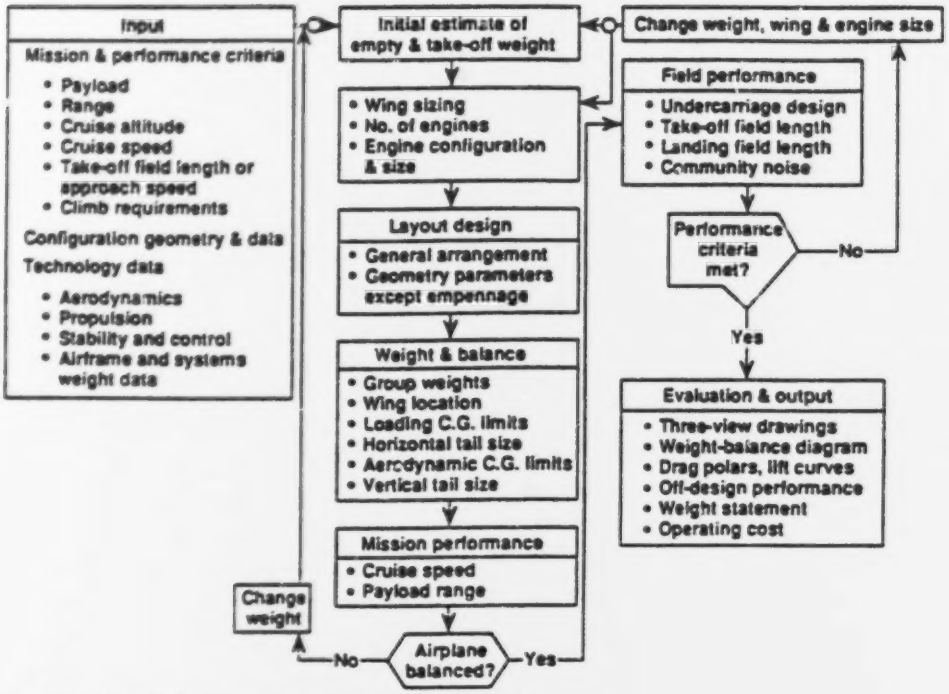


Figure 16. Aircraft design process arranged sequentially.

process whose flowchart reproduced from ref.23 is displayed in Fig.16. The process shown is sequential and indicates in the top, rightmost box that the entire sequence has to be repeated for every change of a design variable. However, the flowchart boxes may be cast as a system of coupled modules shown in Fig.17, with the arrows indicating the transmitted data (examples of data are given in Table 1). Then, the system shown in Fig.17 may be solved for a particular setting of the design variables and analyzed for sensitivity. For the sensitivity analysis, the partial derivatives are computed for each module, and entered into equations analogous to eq.8:

$$\begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 \\ -J_{21} & I & -J_{23} & -J_{24} & -J_{25} & 0 & -J_{27} \\ -J_{31} & 0 & I & 0 & 0 & 0 & 0 \\ -J_{41} & -J_{42} & -J_{43} & I & -J_{45} & 0 & -J_{47} \\ -J_{51} & 0 & 0 & 0 & I & 0 & 0 \\ 0 & -J_{62} & 0 & 0 & -J_{65} & I & 0 \\ -J_{71} & -J_{72} & -J_{73} & -J_{74} & 0 & 0 & I \end{bmatrix} \begin{Bmatrix} dY_1/dX_i \\ dY_2/dX_i \\ dY_3/dX_i \\ dY_4/dX_i \\ dY_5/dX_i \\ dY_6/dX_i \\ dY_7/dX_i \end{Bmatrix} = \begin{Bmatrix} \cdot \\ \cdot \\ \partial Y/\partial X_i \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{Bmatrix} \quad (10)$$

In the above, the numerals refer to the module numbers (in circles) in Fig.17 and the dots in the rhs vector indicate that for a particular design variable many elements of that vector are likely to be null. The incomplete system coupling is reflected in the block-sparseness of the matrix of coefficients.

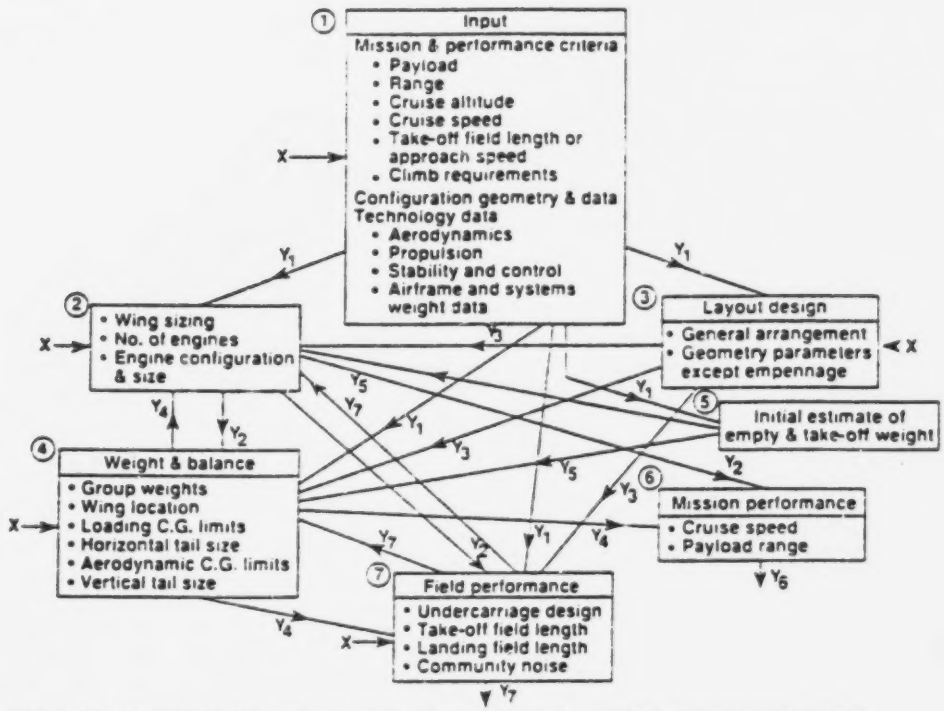


Figure 17. Aircraft design process rearranged into a non-hierarchic, decomposed system.

TABLE 1. EXAMPLES OF COUPLING DATA FOR SYSTEM IN FIG. 17

Vector Y	Content Examples
1	see the box labeled INPUT
2	Wing area, aspect ratio, taper, sweep angle, airfoil geometry data. Engine thrust.
3	Fuel tank locations and assumed volumes.
4	Wing structural weight and internal volume.
5	Take-off Gross Weight.
6	see box 6.
7	Landing gear weight and location, in stowed and extended positions. Take-off field length.

Solution of eq.10 yields the system sensitivity derivatives dY/dXi for as many design variables as many right hand side vectors are included, without the need for repeating the entire sequence shown in Fig.16.

Optimization Guided by System Sensitivity Derivatives

The system sensitivity derivatives may be used by any gradient-guided algorithm to search in design space for an improved design. Experience with that type of optimization has begun to

accumulate. For instance, the effectiveness of such optimization was reported in ref.24 for a simple structure under an impulse load. The structure was enhanced with a control system to limit the transient response, and the system was optimized for minimum weight and minimum control effort in a design space of structural and control design variables.

Encouraging results were also reported in ref.25 for a hypersonic aircraft whose side view is shown in Fig.18. Thermodynamic efficiency of the propulsion in this aircraft is at maximum when the shock wave which emanates from the nose is tangent to the inlet lip as shown in the figure and drops off sharply if the shock wave deviates from this tangency position. The shock wave position is influenced by the shape of the forebody and its tip structural deflection. Since the entire aircraft performance is critically sensitive to the propulsion efficiency, the propulsion and performance couple to aerodynamics and structures. System sensitivity guided optimization of the forebody, simultaneous for aerodynamic shape and structural sizing, proved effective and led to a shape different from the one originally derived as optimal on the grounds of aerodynamics alone for the structure assumed rigid.

Figure 18. Hypersonic aircraft.

DESIGN PROCESS UNIFIED BY SYSTEM SENSITIVITY ANALYSIS

The system sensitivity derivatives may be said to form a mathematical model of design that provides answers to the "what if" questions that pervade the design process. They may also be regarded as means for quantitative communication among the groups of specialists in a design organization, informing how the design decisions in one discipline or subsystem may affect other disciplines and the system as a whole. In general, the information conveyed by the system sensitivity derivatives has not been available at the advanced design stages under the current practice.

When that information is practical to obtain owing to the analysis based on eq.8, it should be possible to organize design process around it in a manner described in ref.8 and depicted in Fig.19. The bubbles labeled with the names of disciplines symbolize groups of specialists using their own computational and experimental tools to output information about the present state of design. Each group's task extends to include computation of the sensitivity derivatives of their output with respect to the inputs received from the other groups and with respect to the design variables, assuming a collective agreement on the cross-fed inputs and design variables. The derivatives are placed as partial derivatives in the framework of the system sensitivity equations (eq.8, 10) from which the system sensitivity derivatives accounting for the inter-disciplinary couplings are obtained.

The disciplinary specialists who contributed the partial derivatives examine the system derivatives to see how the design can be improved by changing the design variables. Chances are that the system sensitivity derivatives will show some design variables as distinctly more influential than others. If so, the desirable changes of the design variables may be apparent, and it may also become obvious how to prune the list of design variables. This is a judgmental use of the system sensitivity derivatives. On the other hand, the derivatives may also be input into a search algorithm

to execute a stage of formal optimization, within bounds guarding against excessive extrapolation errors. The judgmental and formal means of improving the design may reinforce each other. Either way, the decision how to improve the design is made considering simultaneously its impact on the system as a whole and on all the disciplines and subsystems involved.

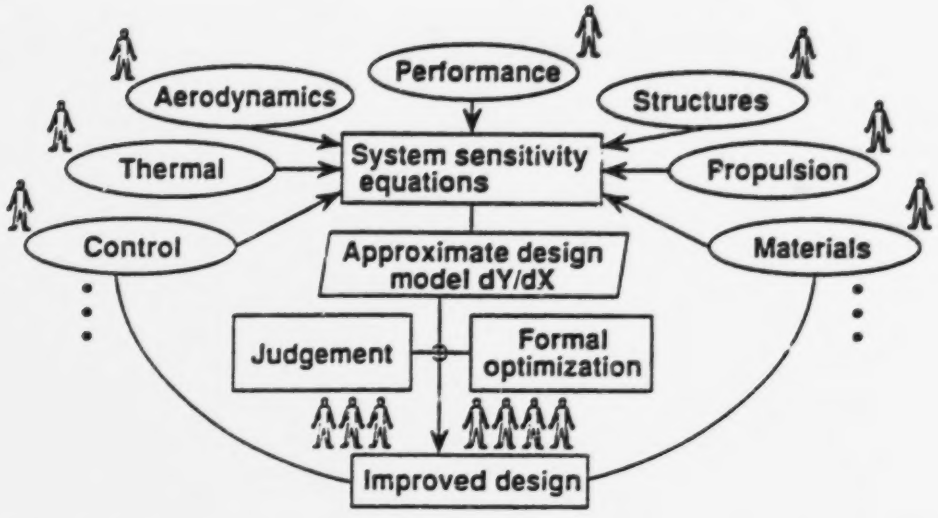


Figure 19. Design process organized around the system sensitivity analysis.

Improving the design will alternate in an iterative loop with updating of the system analysis and sensitivity analysis as required by nonlinearity. The caveat is that the system sensitivity derivatives are meaningful only with respect to the continuous variables of the particular design concept under consideration. They cannot predict the effect of a jump to another discretely different concept (like changing the engine location from under-the-wing to under-the tail), therefore, judgment must be a part of the process. However, when there are competing design concepts, each may be optimized by the above process to reveal its potential for a fair comparison.

The above method may be used at all stages of design process to clearly allocate disciplinary tasks and to compress the schedule by allowing concurrent work on the tasks, while preserving the system couplings with mathematical rigor. Moving to the next more advanced stage of design process would not require any change of the method, only an increase in the analysis depths. Thus, in contrast to the present practice, uniformity of the design process organization would be achieved throughout its stages. Unlike in the currently prevailing sequential design process, the uniform approach would make it possible to retain more design freedom in the later design stages where more is known about the design. The design closer to theoretical optimum should be the ultimate benefit.

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16 Abstract

The currently common sequential design process for engineering systems is likely to lead to suboptimal designs. Recently developed decomposition methods offer an alternative for coming closer to optimum by breaking the large task of system optimization into smaller, concurrently executed and, yet, coupled tasks, identified with engineering disciplines or subsystems. The hierarchic and non-hierarchic decompositions are discussed and illustrated by examples. In conclusion, an organization of a design process centered on the non-hierarchic decomposition is proposed.

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